2013
TRIAL HSC
Examination

Name: ____________________

Teacher: ____________________

Year 12
Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen only
- Board approved calculators may be used
- A table of standard integrals is provided at the back of the paper
- In questions 11 – 16 show relevant mathematical reasoning and/or calculations

Section I ~ Pages 1-5

- 10 marks
- Attempt Questions 1-10
- Allow about 15 minutes for this section
- Use the Multiple Choice Answer Sheet

Section II ~ Pages 6 -14

- 90 marks
- Attempt Questions 11-16
- Allow about 2 hours 45 minutes for this section
- Answer each question in a separate writing book

Teachers:
Mr Bradford
Mr Mulray
Mr Johansen
Ms Yamaner
Ms Ruff

Examiner: Ms Ruff

Write your Name, your Board of Studies Student Number and your Teacher's Name on the front cover of each answer booklet

This paper MUST NOT be removed from the examination room.

Number of Students in Course: 88

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Section I

10 marks
Attempt questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

Question1

Oli and Rohan paid \$315 for a meal at a restaurant. This included a $12\frac{1}{2}\%$ tip. What was the cost of the meal without the tip?

(A) $ 39
(B) $ 276
(C) $ 280
(D) $ 290

Question 2

If $6\sqrt{5}-\frac{1}{\sqrt{5}-2}=a+b\sqrt{5}$, then the values of a and b are:

(A) $a=-2,\ b=-1$
(B) $a=-2,\ b=5$
(C) $a=2,\ b=5$
(D) $a=2,\ b=7$

Question 3

If $\sin\theta=-\frac{2}{3}$ and $\tan\theta<0$, what is the exact value of $\cos\theta$?

(A) $\frac{\sqrt{5}}{3}$
(B) $\frac{2}{\sqrt{5}}$
(C) $-\frac{\sqrt{5}}{3}$
(D) $-\frac{\sqrt{13}}{3}$

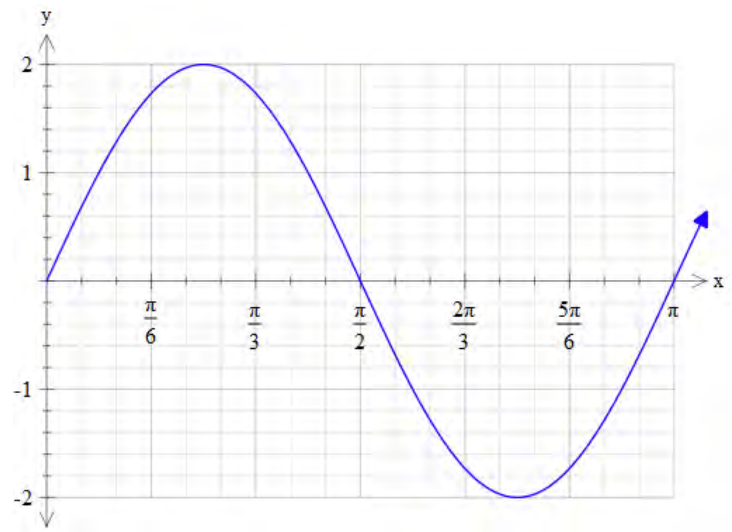
Question 4

What is the domain and range of the function $f(x) = \sqrt{4 - x^2}$?

(A) Domain: $-2 \le x \le 2$ Range: $-2 \le y \le 2$
(B) Domain: $-2 \le x \le 2$ Range: $0 \le y \le 2$
(C) Domain: $0 \le x \le 2$ Range: $-2 \le y \le 2$
(D) Domain: $0 \le x \le 2$ Range: $0 \le y \le 4$

Question 5

The equation of the trigonometric function below could be:



(A) $f(x) = \frac{1}{2}\sin\left(\frac{x}{2}\right)$

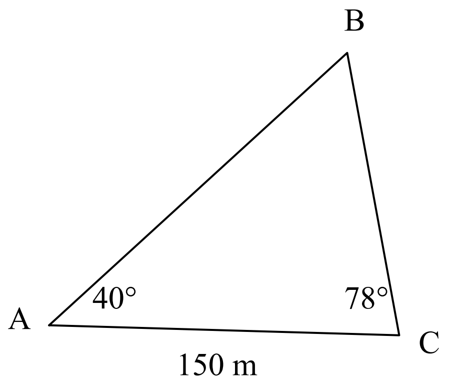
(B) $f(x) = \frac{1}{2}\sin(2x)$

(C) $f(x) = 2\sin\left(\frac{x}{2}\right)$

(D) $f(x) = 2\sin(2x)$

Question 6

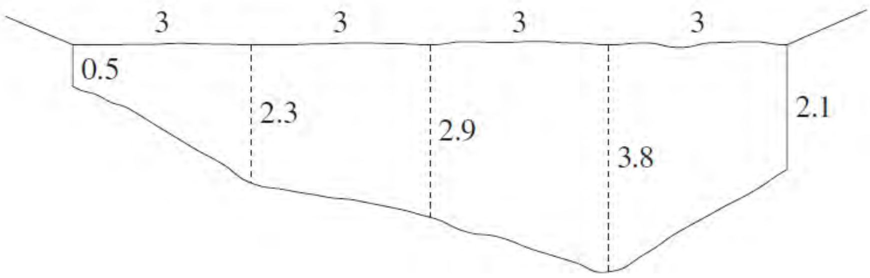
The length of BC correct to the nearest metre is:



(A) 99 m
(B) 109 m
(C) 166 m
(D) 245 m

Question 7

At a certain location a river is 12 metres wide. At this location the depth of the river, in metres, has been measured at 3 metre intervals. The cross section is shown below.

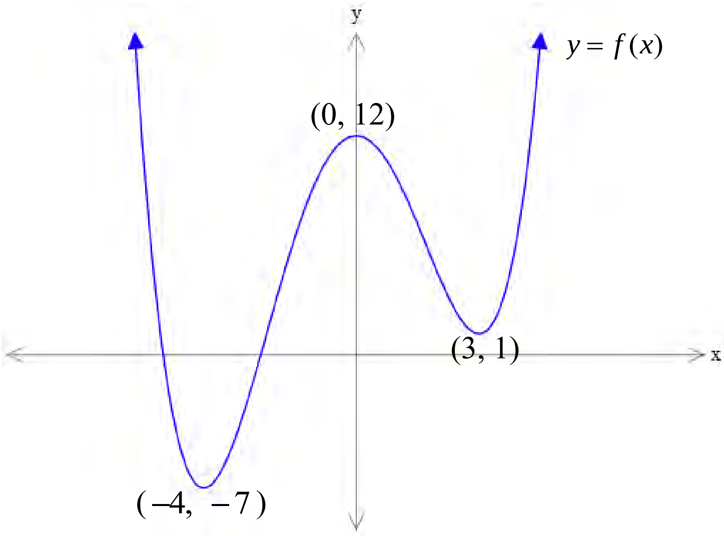


Use the trapezoidal rule with the five depth measurements to calculate the approximate area of the cross section.

(A) 22.6 m^2
(B) 25.2 m^2
(C) 30.9 m^2
(D) 32.8 m^2

Question 8

The diagram shows the graph $y = f(x)$ with stationary points at (0, 12) and (3, 1) and $(-4, -7)$.



For what value of k will $f(x) + k = 0$ have 4 distinct solutions?

(A) $-12 < k < -1$
(B) $-12 < k < 7$
(C) $1 < k < 12$
(D) It is not possible for $f(x) + k = 0$ to have 4 solutions.

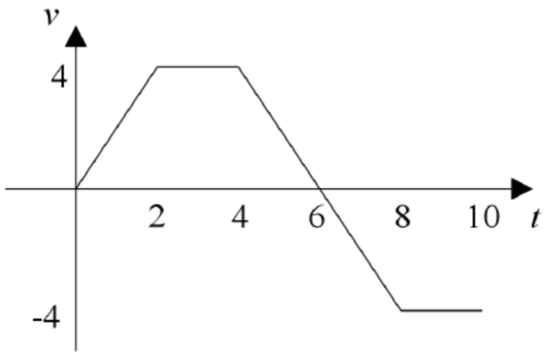
Question 9

A function $y = f(x)$ has $f'(3) = 0$ and $f''(3) = -1$. At the point where $x = 3$, $y = f(x)$ is:

(A) Stationary and concave up.
(B) Decreasing and concave up.
(C) Stationary and concave down.
(D) Stationary with a horizontal point of inflexion.

Question 10

The graph below shows the velocity of a particle for the first 10 seconds of its movement. If the particle is initially 1 m to the left of the origin, where is the particle after 10 seconds?



(A) At the origin
(B) 5 metres to the left of the origin.
(C) 1 metres to the right of the origin.
(D) 3 metres to the right of the origin.

End of Section I

Section II

90 marks
Attempt questions 11 – 16
Allow about 2 hours 45 minutes for this section
Answer each question in a separate writing booklet. Extra writing booklets are available.
All necessary working should be shown in every question.

Question 11 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) If $x^5 = 5000$, find the value of x correct to 3 significant figures. **2**

(b) Simplify $\dfrac{x^3 - 125}{x - 5}$. **2**

(c) Differentiate $x\sqrt{x}$. **2**

(d) An arc length of 5 units subtends an angle θ at the centre of a circle with radius 3 units.
Find the area of the sector. **2**

(e) Find a primitive of $3x^2 - 8$. **1**

(f) A parabola has focus (2, 3) and directrix $y = -1$.
Determine the equation of the parabola. **2**

(g) Evaluate $\int_0^1 (e^{5x} - 1)\,dx$. **2**

(h) In a raffle 30 tickets are sold and there are two prizes.

(i) What is the probability that someone buying 5 tickets does **not** win a prize? **1**

(ii) What is the probability that someone buying 5 tickets wins at least 1 prize? **1**

End of Question 11

Question 12 (15 marks) Use a SEPARATE Writing Booklet. **Marks**

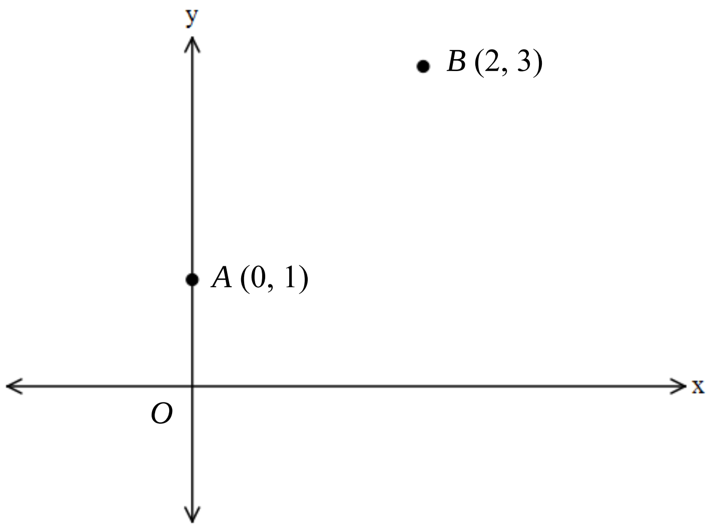
(a) Differentiate with respect to x:

(i) $x \tan 2x$ **2**

(ii) $\dfrac{\ln x}{x}$ **2**

(b) Find $\displaystyle\int \frac{4x}{x^2+6}\,dx$. **2**

(c) Let A and B be the points (0, 1) and (2, 3) respectively.



(i) Find the coordinates of the midpoint of AB. **1**

(ii) Find the gradient of the line AB. **1**

(iii) Show the equation of the perpendicular bisector of AB is $y = 3 - x$. **2**

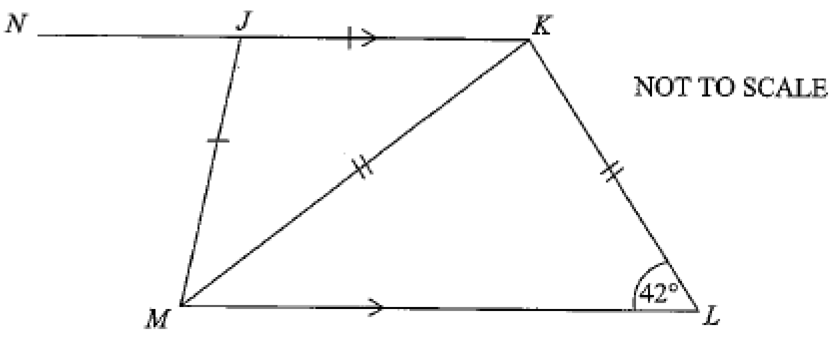
(iv) The point P lies on the line $y = 2x - 9$ and is equidistant from A and B. Find the coordinate of P. **2**

Question 12 continues on page 8

Marks

Question 12 (continued)

(d) The diagram shows a quadrilateral $JKLM$, in which JK is parallel to ML, $JM = JK$, $KM = KL$ and $\angle KLM = 42°$. N is a point on KJ produced.



Find the size of $\angle MJN$. Give reasons for your answer. **3**

End of Question 12

Question 13 (15 marks) Use a SEPARATE Writing Booklet. **Marks**

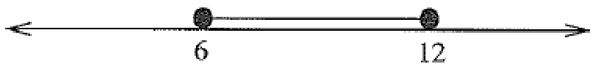
(a) The gradient function of a curve is given by $f'(x) = 3(x+1)(x-3)$ and the curve $y = f(x)$ passes through the point (0, 12).

(i) Find the equation of the curve $y = f(x)$. **2**

(ii) Sketch the curve $y = f(x)$, clearly labeling turning points and the y intercept. **3**

(iii) Assuming there is a point of inflexion at $x = 1$, determine the values of x for which the curve is concave up? **1**

(b) The number line graphs represents the solution to the inequality $|x-a| \le b$.



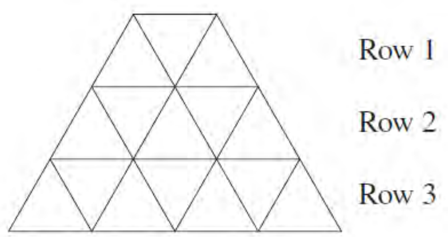
Find the values of a and b. **2**

(c) Find the value of m such that the quadratic equation $x^2 - 4mx + 2m = 0$ has:

(i) one root the reciprocal of the other. **2**

(ii) real roots. **2**

(d) Alex is making a pattern using triangular tiles. The pattern has 3 tiles in the first row, 5 tiles in the second row, and each successive row has 2 more tiles than the previous row.



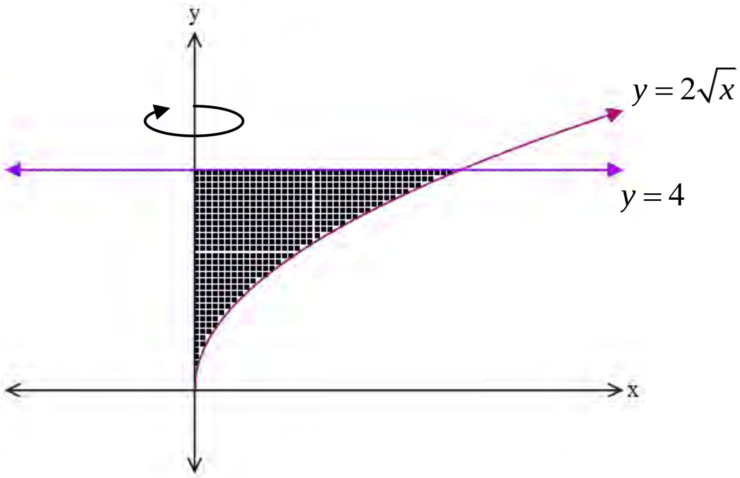
(i) How many tiles would Alex need to use altogether to make the first 20 rows? **1**

(ii) Alex has only 200 tiles. **2**
How many complete rows of the pattern can Alex make?

End of Question 13

Question 14 (15 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) The region bounded by the curve $y = 2\sqrt{x}$, the lines $y = 4$, $y = 0$ and the y-axis is rotated about the y-axis. Find the volume of the solid of revolution formed. **3**



(b) The value of a car depreciates exponentially according to the formula $V = Ae^{-kt}$ where V dollars is the value of the car after t years. A car's new price is \$32 000, and after 6 years its value is \$14 000.

(i) Show that $V = Ae^{-kt}$ satisfies the equation $\dfrac{dV}{dt} = -kV$. **1**

(ii) What is the value of A? **1**

(iii) Find the value of k correct to 3 significant figures. **2**

(iv) Using your answer in part (iii), what will be the value of the car after 10 years? Give your answer to the nearest cent. **1**

(v) At what rate will the value of the car be decreasing when it is 10 years old? **1**

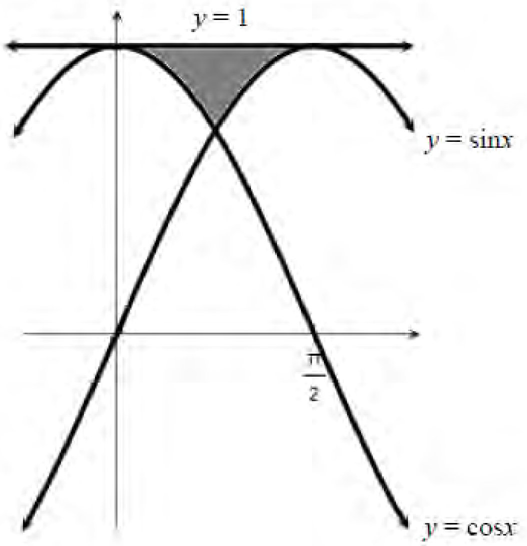
(c) Find the equation of the normal to the curve $y = e^{\cos x}$ at the point where $x = \dfrac{\pi}{2}$. **2**

Question 14 continues on page 11

Marks

Question 14 (continued)

(d) The shaded region in the diagram is bounded by the curves $y = \sin x$, $y = \cos x$ and $y = 1$.



(i) Find the x-coordinate of the point of intersection of $y = \sin x$ and $y = \cos x$. **1**

(ii) Hence or otherwise, find the area of the shaded region. **3**

End of Question 14

Question 15 (15 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) The displacement of a particle is given by $x = t^2 - 4\log_e(t-1) + 5, \quad t > 1$
where x is in metres and t is in seconds.

(i) Find the exact displacement of the particle when $t = 4$. **1**

(ii) Find an expression for v and hence find when the particle comes to rest. **2**

(iii) Show that the acceleration remains positive for $t > 1$. **2**

(iv) Find the exact distance travelled by the particle from the time when the particle comes to rest to $t = 4$. **2**

(b) Find the values of k if $\int_1^k (x+1)dx = 6$ **2**

(c) Michael takes out a loan of \$500 000. The loan is to be repaid in equal monthly repayments, \$$M$, at the end of each month over 30 years (360 months). Reducible interest is charged at 6% per annum, calculated monthly.
Let \$$A_n$ be the amount owing after the nth repayment.

(i) Write down an expression for the amount owing after two months, \$$A_2$. **1**

(ii) Show that the monthly repayment is approximately \$2997.75 **2**

(iii) After how many months will the amount owing, \$$A_n$, become less than \$200 000? **3**

End of Question 15

Question 16 (15 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) (i) The limiting sum of the geometric series: **2**

$3+\frac{3}{\sqrt{3}+1}+\frac{3}{(\sqrt{3}+1)^2}+\ldots$ is of the form $a+b\sqrt{3}$ where a and b are integers. Find a and b.

(ii) Explain why the geometric series **1**

$$3+\frac{3}{\sqrt{3}-1}+\frac{3}{(\sqrt{3}-1)^2}+\ldots$$

does **NOT** have a limiting sum.

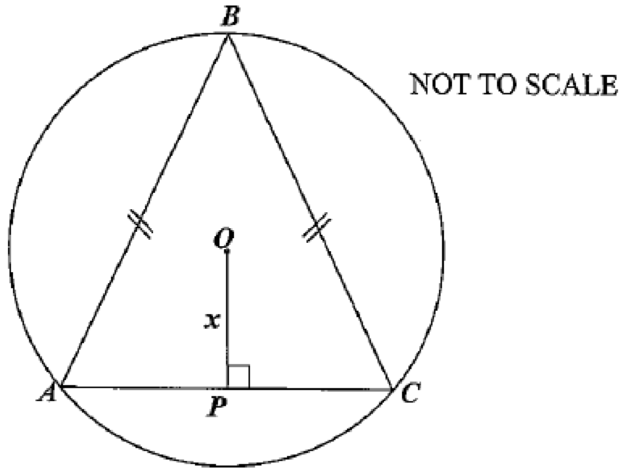
(b) If $x\sec\theta=y\tan\theta$, prove that $\tan\theta\sec\theta=\frac{xy}{y^2-x^2}$. **3**

(c) If the straight line $y=mx$ is a tangent to the curve $y=e^{\frac{x}{2}}$, find the exact value of m. **2**

Question 16 continued on page 14

Question 16 (continued)

(d) An isosceles triangle *ABC*, where *AB* =*BC*, is inscribed in a circle of radius 10 cm. *OP* = *x* and *OP* bisects *AC*, such that *AC* is perpendicular to *OP*.



(i) Show that the area, *A*, of ΔABC is given by $A=(10+x)\sqrt{100-x^2}$. **2**

(ii) Show that $\dfrac{dA}{dx}=\dfrac{100-10x-2x^2}{\sqrt{100-x^2}}$. **2**

(iii) Hence prove that the triangle with the maximum area is equilateral. **3**

End of paper

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STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x \,, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$

2013 Year 12 Mathematics 2 Unit Trial SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
1 C 2 B 3 A 4 B 5 D 6 B 7 C 8 A 9 C 10 D 11a) 5.49 (3 s.f)	1mk ans. 1mk correct rounding	11e) $x^3 - 8$ ✓ $[+C]$ f) vertex $(2,1)$ ✓ $a = 2$ $(x-2)^2 = 8(y-1)$ ✓ g) $\int_0^1 (e^{5x} - 1)\,dx$ $= \left[\frac{e^{5x}}{5} - x\right]_0^1$ ✓ $= \left(\frac{e^5}{5} - 1\right) - \left(\frac{e^0}{5} - 0\right)$ $= \frac{e^5}{5} - 1 - \frac{1}{5}$ $= \frac{e^5 - 6}{5}$ ✓	
b) $\frac{x^3 - 125}{x - 5}$ $= \frac{(x-5)(x^2+5x+25)}{x-5}$ ✓ $= x^2 + 5x + 25$ ✓ c) $x\sqrt{x} = x^{3/2}$ $\frac{d}{dx} x^{3/2} = \frac{3}{2} x^{1/2}$ ✓ $= \frac{3\sqrt{x}}{2}$ ✓ d) $l = 5, r = 3, l = r\theta$ $5 = 3\theta$ $\theta = \frac{5}{3}$ ✓ $A = \frac{1}{2} r^2 \theta$ $= \frac{1}{2} \times 3^2 \times \frac{5}{3}$ $= \frac{15}{2}$ units^2 ✓		h) i) $\frac{25}{30} \times \frac{24}{29}$ $= \frac{20}{29}$ ✓ ii) $1 - \frac{20}{29}$ $= \frac{9}{29}$ ✓	Alt: $\frac{{}^{28}C_5}{{}^{30}C_5}$ $\doteqdot 0.689655$

2013 Year 12 Mathematics 2 Unit Trial SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
12a) i) $x\tan 2x$ $u = x \quad v = \tan 2x$ $u' = 1 \quad v' = 2\sec^2 2x$ ✓ $\frac{d}{dx}(x\tan 2x)$ $= \tan 2x + 2x\sec^2 2x$ ✓ ii) $\frac{\ln x}{x}$ $u = \ln x \quad v = x$ $u' = \frac{1}{x} \quad v' = 1$ ✓ $\frac{d}{dx}\left(\frac{\ln x}{x}\right) = \frac{x \cdot \frac{1}{x} - \ln x \cdot 1}{x^2}$ $= \frac{1 - \ln x}{x^2}$ ✓ b) $\int \frac{4x}{x^2+6}\,dx$ $= 2\int \frac{2x}{x^2+6}\,dx$ $= 2\ln(x^2+6) + C$ ✓ ✓ c) i) $M = (1, 2)$ ✓ ii) $m_{AB} = \frac{3-1}{2-0}$ $= 1$ ✓		iii) $\perp$ bisector $\rightarrow m = -1$ $y - 2 = -1(x - 1)$ ✓ $y - 2 = -x + 1$ $y = 3 - x$ ✓ iv) $y = 2x - 9 \quad -①$ $y = 3 - x \quad -②$ sub ① into ② $2x - 9 = 3 - x$ $3x = 12$ $x = 4$ ✓ when $x = 4$ $y = 3 - 4$ $y = -1$. P has coordinates $(4, -1)$ ✓ d) $\angle KML = \angle KLM = 42$ (angles opposite equal angles of Δ) $\angle JKM = \angle KML = 42$ (alternate angles $JK \parallel ML$) ✓ $\angle JMK = \angle JKM = 42$ (angles opposite equal angles of Δ) $\angle MJN = \angle JMK + \angle JKM$ (exterior angle of Δ) ✓ $= 42 + 42$ $= 84$ ✓	

2013 Year 12 Mathematics 2 Unit Trial SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

13a) i) $f'(x) = 3(x+1)(x-3)$

$= 3(x^2-2x-3)$

$= 3x^2-6x-9$

$f(x) = x^3-3x^2-9x+C$ ✓

$f(0) = 12 = 0^3 - 3\times0^2 - 9\times0 + C$

$C = 12.$

$f(x) = x^3-3x^2-9x+12$ ✓

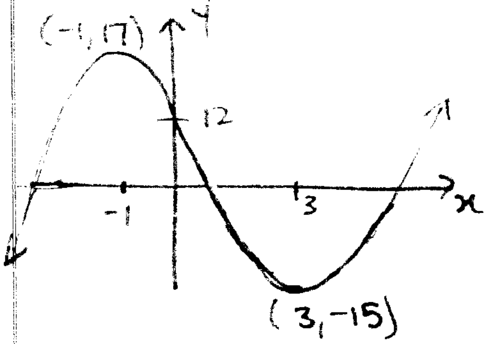
ii) $0 = 3(x+1)(x-3)$

$x = -1, 3$

$(-1, 17) \rightarrow$ local Max

$(3, -15) \rightarrow$ local min.

y-intercept 12.



1mk correct stationary points

1mk correct y-intercept

1mk correctly graphing information

iii) $x > 1$

b) $|x-a| \leq b$

$x - a \leq b$ or $-x+a \leq b$

$x \leq b+a$ $x \geq a-b$

Max value is 12

Min value is 6

$a+b = 12$

$a-b = 6$

$2a = 18$

$a = 9$ ✓

$a-b = 6$

$b = 3$ ✓

c) i) Let the roots be α and $\frac{1}{\alpha}$

$\alpha \cdot \frac{1}{\alpha} = \frac{c}{a}$ ✓

$1 = \frac{2m}{1}$

$2m = 1$

$m = \frac{1}{2}$ ✓

ii) $\Delta \geq 0$ for real roots

$\Delta = b^2 - 4ac$

$= (-4m)^2 - 4\times1\times2m$

$= 16m^2 - 8m$

$16m^2 - 8m \geq 0$ ✓

$2m^2 - m \geq 0$

$m(2m-1) \geq 0$

$m \leq 0, m \geq \frac{1}{2}$ ✓

d) i) $S_{20} = \frac{20}{2}\left[2\times3 + (20-1)2\right]$

$= 440$ ✓

ii) $\frac{n}{2}(3+T_n) = 200$

$T_n = 3 + 2(n-1)$

$= 1+2n$

$\therefore \frac{n}{2}(3+1+2n) = 200$ ✓

$n(4+2n) = 400$

$4n + 2n^2 = 400$

$n^2 + 2n - 200 = 0$

Using quadratic formula

$n = \frac{-2 \pm \sqrt{4+800}}{2}$

$= 13.17\ldots$

$\therefore 13$ ✓ complete rows of the pattern can be made by Alex

Also accept trial & refinement methods.

2013 Year 12 Mathematics 2 Unit Trial SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
14a) $y = 2\sqrt{x}$ $\frac{y}{2} = \sqrt{x}$ $x = \frac{y^2}{4}$ $x^2 = \frac{y^4}{16}$ $V = \pi\int_0^4 \left(\frac{y^4}{16}\right) dy$ ✓ $= \pi\left[\frac{y^5}{80}\right]_0^4$ ✓ $= \pi\left(\frac{4^5}{80} - 0\right)$ $= \frac{64\pi}{5}$ units^3 ✓ b) i) $V = Ae^{-kt}$ $\frac{dV}{dt} = -kAe^{-kt}$ sub $V = Ae^{-kt}$ ✓ $\frac{dV}{dt} = -kV$ as required ii) When $t=0$, $V = 32000$ $32000 = Ae^{-k\times 0}$ $A = 32000$ ✓ iii) when $t = 6$, $V = 14000$ $14000 = 32000e^{-6k}$ $\frac{7}{16} = e^{-6k}$ $\ln\left(\frac{7}{16}\right) = -6k$ ✓ $k = \frac{\ln\left(\frac{7}{16}\right)}{-6}$ $= 0.13777\ldots$ $= 0.138$ (3sf) ✓		iv) when $t=10$ $V = 32000e^{-0.138\times 10}$ $= 8050.51$ $\therefore$ the value of the car after 10 years is \$8050.51 ✓ v) from (i) $\frac{dV}{dt} = -kV$ when $t=10$, $V = 8050.51$ $-kV$ $= -0.138 \times 8050.51$ $= -1110.97$ $\therefore$ The value of the car is decreasing at a rate of \$1111/year (nearest \$) c) $y = e^{\cos x}$ $y' = -\sin x\, e^{\cos x}$ when $x = \pi/2$ $y' = -\sin\left(\frac{\pi}{2}\right) e^{\cos \pi/2}$ $= -1 \times 1$ $= -1$ ✓ when $x = \pi/2$ $y = e^{\cos \pi/2}$ $= 1$ gradient of normal is 1 equation of normal $y - 1 = 1(x - \pi/2)$ $y = x - \pi/2 + 1$ ✓	

2013 Year 12 Mathematics 2 Unit Trial SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
14d) i) $y=\sin x$ $y=\cos x$ $\therefore \sin x = \cos x$. $\frac{\sin x}{\cos x} = 1$ $\tan x = 1$ $x = \frac{\pi}{4}$ ✓ ii) $A = 2\int_0^{\pi/4}(1-\cos x)dx$ $= 2\left[x - \sin x\right]_0^{\pi/4}$ ✓ $= 2\left[\left(\frac{\pi}{4} - \sin\frac{\pi}{4}\right) - (0-\sin 0)\right]$ $= 2\left(\frac{\pi}{4} - \frac{1}{\sqrt{2}} - 0\right)$ ✓ $= \frac{2\pi}{4} - \frac{2}{\sqrt{2}}$ $= \frac{\pi - 2\sqrt{2}}{2}$ units^2 ✓ 15a) $x = t^2 - 4\log_e(t-1) + 5 \quad t>1$ i) when $t=4$ $x = 4^2 - 4\log_e(4-1) + 5$ $= 16 - 4\log_e 3 + 5$ $= 21 - 4\log_e 3$ ✓	Accept $x=\frac{\pi}{4}$ as bald answer	ii) $v = 2t - \frac{4}{t-1}$ ✓ $0 = 2t - \frac{4}{t-1}$ $0 = 2t(t-1) - 4$ $0 = 2t^2 - 2t - 4$ $0 = t^2 - t - 2$ $0 = (t-2)(t+1)$ $t = 2, -1$ But $t=-1$ Not valid $\therefore t = 2$ ✓ particle comes to rest after 2 seconds. iii) $a = 2 + \frac{4}{(t-1)^2}$ ✓ when $t>1$ $(t-1)^2 > 0$ $\therefore 2 + \frac{4}{(t-1)^2} > 0$ Hence $a > 0$ ✓ iv) $\int_2^4 \left(2t - \frac{4}{t-1}\right)dt$ $= \left[t^2 - 4\log_e(t-1)\right]_2^4$ $= (16 - 4\log_e 3) - (4 - 4\log_e 1)$ ✓ $= 12 - 4\log_e 3 + 4\log_e 1$ $= 12 - 4\log_e 3$ ✓ distance travelled is $12 - \log_e 3$ metres	OR $x(4) - x(2)$

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15b) $\int_1^k (x+1)\,dx = 6$ $\left[\frac{x^2}{2}+x\right]_1^k = 6$ $\left(\frac{k^2}{2}+k\right)-\left(\frac{1}{2}+1\right)=6$ $\frac{k^2}{2}+k-1\frac{1}{2}=6$ ✓ $k^2+2k-15=0$ $(k+5)(k-3)=0$ $k=-5, 3$ ✓ c) i) $n=360$ $r=0.005$ $A_1 = 500000\times 1.005 - M$ $A_2 = A_1 \times 1.005 - M$ $=(500000\times 1.005 - M)\times 1.005 - M$ $=500000\times 1.005^2 - 1.005M - M$ ✓ ii) $A_n = 500000\times 1.005^n - M\underbrace{(1+1.005+1.005^2+\dots+1.005^{n-1})}$ GP $a=1$ $r=1.005$ $S_n=\frac{a(r^n-1)}{r-1}$ $=\frac{1.005^n-1}{0.005}$		$A_n = 500000\times 1.005^n - \frac{M(1.005^n-1)}{0.005}$ ✓ when $n=360$, $A_n=0$. $0=500000\times 1.005^{360}-\frac{M(1.005^{360}-1)}{0.005}$ $M = 500000 \times 1.005^{360} \times \frac{0.005}{(1.005^{360}-1)}$ ✓ $= \$2997.75$.	

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15c iii) $A_n = 500000 \times 1.005^n - \frac{2997.75(1.005^n - 1)}{0.005}$ $A_n < 200000$ $200000 > 500000 \times 1.005^n - 599550 \times 1.005^n + 599550$ $-399550 > -99550 \times 1.005^n$ ✓ $\frac{7991}{1991} < 1.005^n$ $\ln\left(\frac{7991}{1991}\right) < \ln 1.005^n$ $\frac{\ln\left(\frac{7991}{1991}\right)}{\ln 1.005} < n$ ✓ $278.66\ldots < n$ After 279 months the amount owing will be less than \$200000 ✓ 16a) $a = 3$ $r = \frac{1}{\sqrt{3}+1}$ $S_\infty = \frac{a}{1-r}$ i) $S_\infty = \frac{3}{1-\frac{1}{\sqrt{3}+1}} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$ ✓ $= \frac{3\sqrt{3}+3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$		16a ii) For limiting sum $-1 < r < 1$ for $3 + \frac{3}{\sqrt{3}-1} + \frac{3}{(\sqrt{3}-1)^2} + \ldots$ $r = \frac{1}{\sqrt{3}-1}$ $\doteqdot 1.366\ldots > 1$ ✓ $\therefore$ no limiting sum	

$= \frac{9+3\sqrt{3}}{3}$

$= 3+\sqrt{3}$

$a = 3$ ✓

$b = 1$

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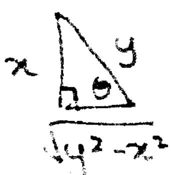
16b) $x\sec\theta = y\tan\theta$

$\frac{x}{\cos\theta} = \frac{y\sin\theta}{\cos\theta}$

$x = y\sin\theta$

$\sin\theta = \frac{x}{y}$

Assuming θ is acute



$\therefore \tan\theta = \frac{x}{\sqrt{y^2-x^2}}$

$\sec\theta = \frac{y}{\sqrt{y^2-x^2}}$ ✓

RTP:

$\tan\theta\sec\theta = \frac{xy}{y^2-x^2}$

$LHS = \tan\theta\sec\theta$

$= \frac{x}{\sqrt{y^2-x^2}} \times \frac{y}{\sqrt{y^2-x^2}}$

$= \frac{xy}{y^2-x^2}$

$= RHS$ ✓

OR

$x^2\sec^2\theta = y^2\tan^2\theta$

$x^2(1+\tan^2\theta) = y^2\tan^2\theta$

$x^2 = (y^2-x^2)\tan^2\theta$

So $\tan^2\theta = \frac{x^2}{y^2-x^2}$

i.e. $\tan\theta\tan\theta = \frac{x^2}{y^2-x^2}$ ✓

Replace one of the $\tan\theta$s by $\frac{x\sec\theta}{y}$

[as $x\sec\theta = y\tan\theta$]

$\Rightarrow \tan\theta \, \frac{x\sec\theta}{y} = \frac{x^2}{y^2-x^2}$ $\otimes$ by $\frac{y}{x}$

$\tan\theta\sec\theta = \frac{xy}{y^2-x^2}$ as required. ✓

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16c) $y = mx$, $y = e^{x/2}$ $\rightarrow e^{x/2} = mx$ — ①

$y' = \frac{1}{2}e^{x/2}$

$\therefore m = \frac{1}{2}e^{x/2}$ — ②

Solving simultaneously

$e^{x/2} = \frac{1}{2}e^{x/2} \cdot x$ — 1mk progress

$1 = \frac{1}{2}x$

$\therefore x = 2$

when $x = 2$

$m = \frac{1}{2}e^{1}$

$m = \frac{e}{2}$ ✓ — 1mk answer

16d) i) $A = \frac{1}{2}bh$

$h = x + 10$

$PC^2 = 10^2 - x^2$

$PC = \sqrt{100 - x^2}$ ✓

$b = 2\sqrt{100 - x^2}$

$A = \frac{1}{2} \times 2\sqrt{100-x^2}\,(x+10)$

$= (10+x)\sqrt{100-x^2}$ ✓

ii) $u = 10 + x$ $\quad v = (100 - x^2)^{1/2}$

$u' = 1$ $\quad v' = \frac{1}{2}(100 - x^2)^{-1/2} \cdot -2x$

$= \frac{-x}{\sqrt{100 - x^2}}$ ✓

$\frac{dA}{dx} = \sqrt{100 - x^2} + \frac{(10+x).(-x)}{\sqrt{100-x^2}}$

$= \frac{100 - x^2 - 10x - x^2}{\sqrt{100-x^2}}$

$= \frac{100 - 10x - 2x^2}{\sqrt{100-x^2}}$ ✓

iii) Max will occur when $\frac{dA}{dx} = 0$

$0 = \frac{100 - 10x - 2x^2}{\sqrt{100-x^2}}$

$0 = 100 - 10x - 2x^2$

$0 = x^2 + 5x - 50$

$0 = (x+10)(x-5)$

$x = -10, 5$ $\quad$ -10 not valid

$\therefore x = 5$ ✓

check

x	4	5	6
$\frac{dA}{dx}$	$1/3$	0	$-1/2$

✓

Max.

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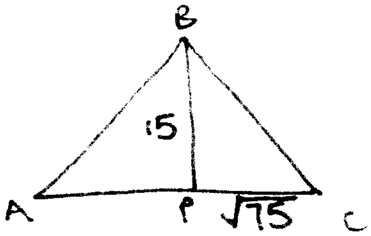
when $x = 5$  $AC = 2PC$ $= 2\sqrt{75}$ $= 10\sqrt{3}$ $BC^2 = 15^2 + (\sqrt{75})^2$ $= 300$ $BC = \sqrt{300}$ $= 10\sqrt{3}$. $AB = BC$ (given) $\therefore AB = 10\sqrt{3}$. $\therefore AB = BC = AC$. $\therefore \Delta ABC$ has maximum area when it is equilateral.			